MODIFIED COEFFICIENT OF SUBGRADE REACTION TO LATERALLY LOADED PILES

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Abstract

An accurate prediction of the load capacity of a laterally loaded pile at a permissible displacement is an important concern at the design stage. In contrast to many sophisticated methods, Broms' method based on moment equilibrium has been preferred by engineers to predict the load capacities of laterally loaded piles due to both its simplicity and because it is established on a way of hand calculation. However, Broms' method typically overestimates a pile's lateral load capacity as it requires a constant coefficient of horizontal subgrade reaction (n_h) into analyses, regardless of the magnitude of the pile's top displacement. In this study, modified coefficients of subgrade reactions (n_h^) that are sensitive to the pile's top displacement in cohesionless soils are first proposed to improve the performance level of Broms' method for the prediction of the load capacity of a laterally loaded pile as the pile's top displacement increases. The modified values of n_h^* are calibrated using 45 independently free-head, single-driven, full-scale pile tests from 23 sites in cohesionless soils. It is demonstrated that Broms' method with n_h^* would correctly estimate a pile's lateral load-deflection behavior with accuracy levels similar to more complicated methods.*

1 INTRODUCTION

The failure load of a laterally loaded pile cannot be easily defined by a predefined limit unless the pile fails structurally. Therefore, a prediction of the load of a laterally loaded pile remains largely unknown as it requires the systematic examination of the measured performance of many full-scale loaded piles under lateral loads and site conditions.

The load capacities of laterally loaded piles at a given lateral displacement have been easily estimated with the help of Broms' method, which uses simple equations based on the moment equilibrium utilizing the elastic theory along with the coefficient of subgrade reactions to calculate the pile's top displacement at the ground surface. An important parameter that affects the performance level in the prediction of a load at a pile's top displacement for Broms' method is the coefficient of horizontal subgrade reaction (n_h) developed by Terzaghi [1], based on the soil's relative density and the location of the layer above or below the ground water table, and presented in Table 1.

Broms' method typically overestimates a pile's lateral load capacity at a given pile's top displacement due to the implementation of a constant value of n_h into the analyses of Broms' method, regardless of the magnitude of the pile's displacement [2]. It has been reported, however,

Table 1. Approximate values of the coefficient of subgrade reaction (n_h) recommended by Terzaghi [1], varying with the relative density of the soil and the location of the ground-water table.

Relative Density	n_h (kN/m ³)		
	Loose	Medium	Dense
Above Ground-Water Table (GWL)	2424	7272	19393
Below GWL	1385	4848	11774

that the coefficient of subgrade reaction decreases as the pile displacement increases [2, 3, 4, 5 and 6]. Researchers [7, 8, 9, 10, 11, 12 and 13] investigated the effect of the subgrade reaction coefficient of a soil deposit using a back analysis based on deflection data from horizontal loading tests. Nevertheless, the coefficient of the horizontal subgrade reaction, an important parameter for Broms' method, has been broadly studied by researchers, there has been only an inadequate full-scale field verification for it. In this study, modified coefficients of subgrade reactions (n_h^*) that vary with an increase in the pile's top displacement in cohesionless soils are suggested to enhance the performance level of Broms' method for the prediction of a lateral load capacity of the piles as the pile's top displacement increases. Hence, Broms' method would be compatible with more complex methods such as the p-y curve method and the Strain Wedge Method (SWM). The proposed modifications are calibrated by the cases of 45 free-head, single-driven, full-scale pile tests that are gathered from 23 sites in cohesionless soils to determine the performance level in the prediction of a load (P_{cal}) required to induce a displacement from 12.7 to 63.5 mm at 12.7 mm intervals with an actual load (P_{msd}) measured at the same displacements. It is confirmed that Broms' method with values of n_h^* would properly estimate the pile's lateral load-deflection behavior at precision levels similar to the p-y curve method and the Strain Wedge Method (SWM).

2 ANALYSES METHODS

The methods of analysis for pile behaviour under lateral loads range in complexity from simple empirical methods to three-dimensional finite-element methods. The 45 free-head, single, full-scale pile tests in cohesionless soils were assembled in this study and examined mostly via the approach seeking the performance level in the calculation of a P_{calc} required to induce a displacement from 12.7 to 63.5 mm at 12.7 mm intervals with the actual loads measured (P_{msd}) at the same displacements using Broms' method [14, 15 and 16]. Then, the analysis results of Broms' method were compared with the p-y curves [17 and 18] and the Strain Wedge Model (SWM) [19, 20 and 21].

Broms [14 and 15] separated the analysis into cases of laterally loaded piles that are embedded in cohesive and cohesionless soils. Broms also suggested different procedures for the prediction of laterally loaded piles under working loads and for an assessment of the pile's ultimate resistance. In the working state, Broms [14 and 15] assumed that under loads of less than one-half to one-third of the ultimate lateral resistance, the deflections of a single pile increased approximately linearly with the applied load. Subsequently, the unit soil reaction (p) acting on a laterally loaded pile increased in proportion to the lateral deflection (y). Broms' solution for the deflection under the application of small loads is based on the beam in elastic foundation theory.

Broms categorized the piles as either long or short piles, the dimensionless depth of embedment is defined as ηL , where $\eta = \sqrt[5]{n_h/EI}$. Accordingly, the depth of the embedment for a long pile would be $\eta L > 4$ and $\eta L < 2$ for a short pile. The surface deflection of a single pile can be obtained by using equation 1 or 2 based on the moment equilibrium for a free-head driven short pile or a long pile, respectively. Additional solutions for the surface displacement of the piles were presented by Broms (14, 15, 16).

$$Y_0 = \frac{18P(1 + 1.33e/L)}{L^2 n_h} (\eta L \leq 2) \quad (1)$$

$$Y_0 = \frac{2.40P}{(n_h)^{3/5} (EI)^{2/5}} (\eta L \geq 4) \quad (2)$$

where P is the applied load at the pile's top, EI is the stiffness of the pile section, L is the embedded length of the pile, e is the load eccentricity and Y_0 is the pile's top displacement.

The lateral loads in many cases are moderate to relatively high, for which nonlinearity typifies the pile's load-deflection behavior. As a result, the nonlinear elasticity methods have been developed in which the application of elastic solutions for the equivalent soil properties is used in an iterative procedure, ending when the displacement compatibility between the soil and the pile is achieved. The most commonly used such method is referred to as the p-y curves method, first devised by McClelland and Focht [22] and improved over the years by others. In the p-y curves procedure, nonlinear curves relating the soil reaction (p) to the pile displacement (y) at various depths are first constructed. An initial stiffness of the soil is assumed, and the governing elastic differential equation of the pile's deflection is solved numerically for that stiffness, resulting in a distribution of the pile's deflection. With the calculated deflection, entering the p-y curves, the soil reaction can be evaluated and compared with the one initially assumed. If a substantial difference exists, the soil stiffness at the depth for which

the curve was drawn is taken as the secant modulus from the p - y curve at the previously computed deflection and the procedure is then repeated until a tolerable compatibility is reached. Not exactly true, the Beam on Elastic Foundation (*BEF*) assumption at the base of the solution has been shown to produce accurate results for the patterns of the deflections that can occur in practice [17]. Subsequently, three matters need to be considered: (i) the elastic analysis of the interaction between the soil and the pile, modeled using the concept of a subgrade reaction, (ii) the estimation of the shape of the p - y curves for various types of soil and loading conditions, and (iii) the procedure in which the nonlinearity is incorporated into the solution. A comprehensive review of the historical development of research on those three issues has been provided by Duncan et al. [23]. According to the subgrade reaction approach, the laterally loaded pile is treated as a *BEF*. Winkler's [24] soil model is assumed, in which the elastic soil medium is replaced by a series of infinitely closely spaced independent and elastic springs with a stiffness equal to the modulus of the horizontal subgrade reaction (k_h)

$$k_h = \frac{p}{y} \quad (3)$$

$$EI \times \frac{d^4 y}{dz^4} + p = 0 \quad (4)$$

where EI is the pile stiffness, and p is the soil reaction that is equal to $k_h y$. Thus, the governing differential equation can be rewritten as follows:

$$EI \left(\frac{d^4 y}{dx^4} \right) + E_s(x) = 0 \quad (5)$$

The deflection of the pile at each depth is dependent on several parameters, the depth, the relative stiffness factor that combines the stiffness of the pile and the soil and their interaction, and the type and magnitude of the applied load at the pile's top. The solution to the governing equation involves specifying the four boundary conditions, such as the known values of the shear force, the moment, the slope or the deflection at both ends of the pile. Details of the solution based on the dimensional analysis are described by Matlock and Reese [25], Prakash and Sharma [26], and Poulos and Davis [27]. The result of solving the governing equation yields five distributions with the pile depth: deflection (y), slope ($S=dy/dz$), bending moment ($M=EI d^2 y/dz^2$), shear ($V=EI d^3 y/dz^3$), and soil reaction ($p=EI d^4 y/dz^4$). The application of the p - y curves' analysis requires the use of computer codes to assess the deflections of the pile and the bending moments it develops under various loads. Relevant programs have been developed by Reese [28, 29].

The Strain Wedge Model (*SWM*) is an approach that has been developed to predict the response of a pile under lateral loading [19]. It can be applied to both driven piles and in-place-constructed deep foundations. The main concept of the *SWM* is that traditional one-dimensional *BEF* pile response parameters can be characterized in terms of the three-dimensional soil-pile interaction behavior. The *SWM* parameters are related to an envisioned three-dimensional passive wedge of soil developing in front of the pile [21]. As a result, the *SWM* is able to provide a theoretical link between the more complex three-dimensional soil-pile interaction and the simpler one-dimensional *BEF* characterization, hence allowing the appropriate selection of the *BEF* parameters to solve the fourth-order ordinary differential equation with the modulus of soil subgrade reaction (E_s) being associated with the *BEF* characterization:

$$EI \left(\frac{d^4 y}{dx^4} \right) + E_s(x) = 0 \quad (6)$$

The closed-form solution to equation 6 was obtained by Matlock and Reese [25] for the case of uniform soil. The governing analytical formulations should be related to the passive wedge in front of the pile, the soil's stress-strain relationship, and the related soil-pile interaction. It should be noted that the *SWM* is based on an effective stress analysis of both sand and clay. The computer code of the *SWM* was used to analyze the presented case histories.

3 DATABASE AND SOIL PARAMETERS

The database gathered for this study includes the reported case histories from 23 sites with 45 laterally loaded, free-head, single driven piles in cohesionless soils including 19 H piles (*HP*) at 11 sites, 20 pipe piles (*PP*) at 8 sites, and 6 precast-prestressed-concrete piles (*PPCP*) at 4 sites.

The case histories were selected only if they had load-settlement curves that extend at least beyond the serviceability criteria of 38.2 mm [30]. The pile length, the size, and the eccentricity of the applied load and the ground-water table (*GWL*) are summarized in Table 2. Detailed information about each loaded test pile was given in Gurbuz [2].

For the majority of the case histories, most of the boring logs contained the Standard Penetration Test blow counts (*SPT-N*) that were the only in-situ soil-property measurement. With the aim of maintaining consistency to the best possible extent, whenever *SPT-N* values were available, they were used to establish the soil parameters

Table 2. Database for laterally loaded free-head single piles and a general description of the soil.

Pile type	Counter No	Volume No	Pile Site	Pile No	Pile Size (mm x mm)	Embedded length (L_p) (m)	Eccentricity (e) (m)	GWL (m)
H Pile	1	1	1	14	310×110	14.9	0.3	-
H Pile	2	1	1	15	310×110	15.2	0.3	-
H Pile	3	2	2	13	310×110	18.4	0.3	-
H Pile	4	3	3	2	250×62	7.4	0.3	-
H Pile	5	3	3	3	250×62	7.3	0.3	-
H Pile	6	3	4	2	250×62	10.1	0.3	-
H Pile	7	3	4	3	250×62	9.7	0.3	-
H Pile	8	45	21	6	360×108	23.0	0.3	1.6
H Pile	9	45	35	5	310×110	27.6	0.3	9.0
H Pile	10	45	37	3	310×79	14.5	0.6	1.5
H Pile	11	45	37	4	310×79	39.0	0.3	1.5
H Pile	12	45	37	5	310×79	31.2	0.3	1.5
H Pile	13	45	37	6	310×110	14.5	0.4	1.5
H Pile	14	45	37	7	310×110	45.3	0.3	1.5
H Pile	15	45	37	8	310×110	30.9	0.3	1.5
H Pile	16	45	39	2	310×110	25.5	0.2	3.0
H Pile	17	45	40	2	310×110	24.5	0.2	5.0
H Pile	18	45	41	2	310×110	19.5	0.2	7.6
H Pile	19	84	BOC	P10	310×110	21.3	0.9	0.5
PP CE	20	2	2	8	323.85×9.53	18.3	0.3	-
PP CE	21	2	2	10	323.85×9.53	18.3	0.3	-
PP OE	22	29	-	PP	406.40×12.70	20.4	0.3	-
PP CE	23	45	35	6	324.10×6.30	27.4	0.2	6.9
PP CE	24	45	39	3	324.10×6.30	25.4	0.2	3.8
PP CE	25	45	40	3	324.10×6.30	17.2	0.2	5.0
PP OE	26	FHWA	4	1	475.20×6.35	24.4	0.3	-
PP OE	27	FHWA	4	2	457.20×6.35	24.4	0.3	-
PP OE	28	88	1	A	609.60×19.05	10.1	0.5	4.7
PP OE	29	88	1	B	609.60×19.05	8.6	0.5	4.7
PP OE	30	88	1	C	609.60×19.05	8.5	0.4	4.7
PP OE	31	88	1	D	609.60×19.05	9.8	0.4	4.7
PP OE	32	88	1	M	609.60×19.05	9.1	0.5	4.7
PP OE	33	88	1	P	609.60×19.05	9.1	0.5	4.7
PP OE	34	88	1	R	609.60×19.05	8.5	0.4	4.7
PP OE	35	88	1	T	609.60×19.05	8.5	0.4	4.7
PP OE	36	88	1	U	609.60×19.05	8.5	0.3	4.7
PP OE	37	88	1	W	609.60×19.05	8.5	0.4	4.7
PP OE	38	88	3	C	1066.80×19.05	30.6	0.6	2.1
PP OE	39	88	3	H	1066.80×19.05	30.6	0.6	2.1
PPC S	40	55	A	24E	609.60×609.60	16.3	4.9	4.3
PPC S	41	55	A	24W	609.60×609.60	16.6	4.9	4.3
PPC S	42	55	B	30EM	762.00×762.00	13.1	4.7	4.3
PPC S	43	55	B	30	762.00×762.00	13.7	4.7	4.3
PPC S	44	RI	1	PPC1	355.60×355.60	27.0	0.4	3.1
PPC S	45	RI	2	PPC3	355.60×355.60	35.7	0.6	3.1

H: H pile, OE: open-ended piles, CE: close-ended piles, ϕ : diameter, S: square

Table 3. Correlations of the soil parameters from the Standard Penetration Test (SPT-N) blow counts.

	Property	Correlation	Reference
Cohesionless Soil	Internal friction angle	$\phi(^{\circ}) = 27.1 + 0.3 \times (N_1)_{60} - 0.00054[(N_1)_{60}]$	Peck et al., [34]
	Corrected SPT value	$(N_1)_{60} = \left(\frac{P_a}{\sigma'}\right)^{0.5} \times N$	Liao and Whitman [35]
	Soil modulus of elasticity	$E_s/P_a = 200 \times \ln(N_{60}) \quad N \leq 60$	Present study

following the relevant procedures used in the LRFD calibration of Deep Foundations, NCHRP Report 507 [31]. A detailed development of the soil parameters was outlined in the work carried out by Gurbuz [2]. Reported well-known equations for determining the soil properties in this study, also used by many researches around the world, are presented in Table 3.

4 ANALYSES OF THE METHODOLOGY AND RESULTS

The failure load of a laterally loaded pile cannot be easily defined by a predefined limit unless the pile fails structurally. As such, the serviceability of bridges, which deals with the functionality and service requirements of a structure to ensure adequate performance under expected conditions, is defined, among other ways, by the maximum possible lateral displacement. AASHTO [32] and previous specifications are based on Moulton [33], limiting the total lateral displacement of bridge substructures to 38.2 mm and, if combined with vertical displacement, to 25.4 mm (Paikowsky and Lu, 2006). The case histories were selected only if they had load-settlement curves that extend at least beyond the serviceability criteria of 38.2 mm [30]. Subsequently, 45 free-head, single, full-scale pile tests in cohesionless soils from 23 sites were examined for a displacement up to 63.5 mm at 12.7 mm intervals, while the analysis results of the Broms' method were compared with the p-y curves and the SWM.

The accuracy level in the prediction of P_{cal} over P_{msd} at a given displacement for each of the individual laterally loaded piles in this study is represented in terms of the bias value (λ_i), the mean of the bias (λ_m) and the coefficient of variation ($COV = \mu$). λ_i , λ_m , the standard deviation (σ), and the COV, which were calculated from the ratios of P_{msd} to P_{cal} at the same vertical load for a given number of tests (N) to ascertain the performance levels of the calculated loads from the analyses methods at a given lateral displacement as follows:

$$\lambda = \frac{P_{msd}}{P_{cal}} \quad (7)$$

$$\lambda_m = \frac{\sum \lambda_i}{N} \quad (8)$$

$$\sigma = \sqrt{\frac{1}{N-1} \sum (\lambda_i - \lambda_m)^2} \quad (9)$$

$$COV = \frac{\sigma}{\lambda_m} \quad (10)$$

The most important parameter that has been extensively studied by researchers is the coefficient of the horizontal subgrade reaction (n_h) for Broms' method, nevertheless, there has only been limited full-scale field verification for it. Accordingly, 45 free-head, single, full-scale pile tests in cohesionless soils from 23 sites were examined mostly via the approach seeking the performance level in the calculation of P_{calc} required to induce a displacement from 12.7 to 63.5 mm at 12.7 mm intervals with P_{msd} at the same displacements using Broms' method [14,15 and 16] with constant values of n_h . Then, the analysis results of Broms' method were compared with both the analyses results of the p-y curves [17 and 18] and the SWM [19,20 and 21]. The predictions of P_{calc} required to induce displacements with P_{msd} at the same displacements for the pile cases in cohesionless soils from the analyses methods of Broms with constant values of n_h , the p-y curves and the SWM were calculated and the analysis results of the three methods are presented in Table 4 in terms of λ_m and COV. The interpreted results show that the values of λ_m for the p-y curves and the SWM decrease almost linearly and later stabilize around 1.0; however, the λ_m of Broms' method declines drastically and becomes less than 1.0 for the analyzed pile types as the pile's top displacement increases. The differences in λ_m between the Broms' method and the two aforementioned methods are in the range 7 % to 57% due to the implemented constant values of n_h into the analyses of Broms' method.

Based on the analysis results of Broms' method in Table 4, it can be concluded that Broms' method needs a calibration for the constant values of n_h , which are independent of the pile's top displacement. in Table 1. Hence, the cases of 45 laterally loaded free head of single piles in cohesionless soils were re-analyzed to assess the modified coefficients of the subgrade reaction (n_h^*) which were sensitive to the pile's displacement, while the bias

Figure 1. Calculated coefficient of subgrade reaction for:
(a) loose soil, (b) medium soil, (c) dense soil located either above the GWL or below the GWL.

Table 4. Summary of the statistics associated with the evaluation of the lateral displacement of the pile types using the methods of analysis for all the pile types.

Pile Type	Analysis method	Pile Top Lateral Deflection (mm)									
		12.7		25.4		38.1		50.8		63.5	
		λ_m	μ	λ_m	μ	λ_m	μ	λ_m	μ	λ_m	μ
HP	p-y	0.993	0.266	1.031	0.261	1.026	0.256	1.021	0.177	1.034	0.165
	Broms	1.105	0.245	0.927	0.255	0.812	0.292	0.648	0.164	0.591	0.192
	SWM	0.944	0.250	0.985	0.224	0.984	0.224	0.945	0.196	0.946	0.188
PP	p-y	1.121	0.295	1.077	0.277	1.094	0.238	0.981	0.283	1.053	0.229
	Broms	1.179	0.197	0.929	0.173	0.805	0.173	0.805	0.231	0.715	0.134
	SWM	1.021	0.385	1.000	0.358	0.995	0.319	0.979	0.418	1.252	0.274
PPC	p-y	1.536	0.244	1.211	0.165	1.065	0.139	0.981	0.119	0.908	0.119
	Broms	1.127	0.156	0.788	0.185	0.616	0.132	0.525	0.107	0.447	0.111
	SWM	1.889	0.156	1.504	0.104	1.310	0.093	1.212	0.094	1.132	0.101

value (λ_i) of each pile case for the ratios of P_{msd} to P_{cal} at a given pile's top displacement was assumed to be equal to one. The back calculations of the values of n_h^* with one standard deviation from the back analyses of Broms' method for loose, medium and dense cohesionless soils under both above and below GWL are plotted in Figure 1. The determined equations yield to average values of n_h^* with the function of the displacement are furnished in the following equations 11 through 16.

$$n_h^* (\text{kN/m}^3) = 8700 * Y_0^{-0.40} \text{ for loose soils above GWL} \quad (11)$$

$$n_h^* (\text{kN/m}^3) = 3716 * Y_0^{-0.60} \text{ for loose soils below GWL} \quad (12)$$

$$n_h^* (\text{kN/m}^3) = 33000 * Y_0^{-0.50} \text{ for medium dense soils above GWL} \quad (13)$$

$$n_h^* (\text{kN/m}^3) = 5000 * Y_0^{-1.00} \text{ for medium dense soils below GWL} \quad (14)$$

$$n_h^* (\text{kN/m}^3) = 123000 * Y_0^{-0.70} \text{ for dense soils above GW} \quad (15)$$

$$n_h^* (\text{kN/m}^3) = 165000 * Y_0^{-1.10} \text{ for dense soils below GWL} \quad (16)$$

The database of 45 laterally loaded, free-head driven, single piles in cohesionless soils was re-used for the prediction of P_{cal} required to induce the displacement from 12.5 mm to 63.5 mm at 12.7 mm intervals with P_{msd} measured at the same displacement using Broms' method with both values of the n_h^* (Fig. 1, and equation 11 through 16) and n_h in Table 1. The analysis results for all the pile types obtained from Broms' method with values of both n_h and n_h^* were compared with the analysis results of the p-y curve and the SWM and presented in Figure 2. The analysis results of the pile cases showed that the λ_m of Broms' method stabilized around 1.0 for all the pile types for any given displacement if the values of n_h^* were employed in the analyses of Broms' method.

Figure 2. Mean of the bias and the coefficient of variation of the ratio between the measured and calculated load from the analyses methods for all the pile types together as the pile's top displacement varies between 12.5 to 63.5 mm.

5 SUMMARY AND CONCLUSIONS

A precise calculation of the load capacity for a given pile's top displacement is vital for the design of laterally loaded piles. The load capacities of laterally loaded piles in this study at a given displacement were

easily calculated using Broms' method with constant values of the coefficient of the subgrade reaction (n_h). Nevertheless, it was determined that Broms' method typically overemphasizes the pile's lateral load capacity due to an implementation of the constant values of n_h into analyses of Broms' method, nevertheless the pile's displacement increases. Hence, modifications to the values of the coefficient of the subgrade reaction (n_h^*) in cohesionless soils that vary with a pile's top displacement are proposed and improved the performance level of the Broms' method for the prediction of a lateral load at the pile's top displacement. Therefore, the overall behavior of all the piles is analyzed by Broms' method with the modified coefficient of the subgrade reactions (n_h^*) yielded to the mean of the bias value of 1 at a given lateral displacement up to 63.5 mm of a pile top's displacement. It is demonstrated that Broms' method with n_h^* can correctly estimate the pile's lateral load-deflection behavior at accuracy levels similar to both the p-y curve method and the Strain Wedge Method.

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