EFFECT OF PARTICLE ROUNDNESS AND MORPHOLOGY ON THE SHEAR FAILURE MECHANISM OF GRANULAR SOIL UNDER STRIP FOOTING

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Abstract

This study investigates the effect of particles roundness and morphology on the shear failure mechanism of soil. A strip footing was modeled under laboratory conditions. Calcareous soil was tested with three roundness classes: angular, rounded and well-rounded shapes with sizes of 0.30 mm to 4.75 mm. These were divided into six different groups at three relative densities of 30%, 50% and 70%. A series of photographs was taken during the tests and analyzed using the particle image velocimetry (PIV) method to understand the soil-deformation mechanism. The results showed that increasing the sample sizes increased the affected area of the soil. At the same time, increasing the relative density caused a punching failure mechanism that went towards the general failure. The shear failure mechanism of the soil changed from general toward punching shear failure with increasing particle roundness. This effect was larger with the smaller materials. Underneath the affected layers of soil, the angular samples were deeper than the rounded and well-rounded samples. The affected depth in the angular soil was approximately 1.5B in the smallest size group. This was more than 3B and near 4B in the largest size group. Both the sides and the underlying soil layers should be considered on angular soils. The area under the footing becomes more important than the side parts after increasing the roundness of the particles.

1 INTRODUCTION

Soil's geotechnical properties primarily affect its behavior. One of these properties is particle shape. The texture, sphericity, roundness and roughness are used to describe the particle shapes [1, 2]. Holtz and Gibbs [3] showed that the shear strength of angular materials is more than the rounded or well-rounded materials. Other studies showed that by increasing the angularity of the materials, the maximum and minimum void ratios (e_{max} and e_{min}) decreased [2, 4, 5]. There are many studies on size or particle shape and their effect on the behavior of aggregates under loading. Direct shear box tests have been made on samples prepared with groups of natural river soil and crushed gravels and fine soil including clay and silt. Yanrong studied the effect of size distribution and particle shape [6].

Arasan et al. [7] studied calcareous, ballast, abrasive and bearing balls. The shape properties of the particles were calculated by considering the roundness values, and the soils were divided into six roundness classes. Calcareous soil changed from angular to rounded and well-rounded, and the effect of the particle shapes on the geotechnical properties of the aggregate was studied.

The ultimate bearing capacity of the strip footing is one of the most important issues in civil engineering [8]. Many researches and investigations have been made on strip footing. Reinforcing the soil can improve the settlement of the footing or bearing capacity of the soil [9, 10, 11, 12, 13, 14]. Different kinds of soils such as gravel, sandy soil, clay and silt under different conditions in normal or reinforced soil have been studied. Previous researchers have studied different conditions and the effect of different factors such as the shape of the footing, soil properties, reinforcement, ground-water level, etc. on the manner of the strip footing [15, 16, 17, 18].

Chen and Abu-Farsakh [19] studied the strip footing and ultimate bearing capacity of the reinforced soil. They developed an analytical solution for estimating the ultimate bearing capacity. The results showed that relative density of reinforced soil and underlying un-reinforced soil affected punching shear failure. Kuranchie et al. [20] studied load-settlement behavior of strip footing laid on iron ore tailings. Cicek et al. [21] studied reinforcing soil and the effect of reinforcement length on the strip footing behavior by geogrids. Reinforcement length as well as the types and number of reinforcements were tested to determine whether they affect the optimum reinforcement.

The effect of the ultimate load has also been studied [22]. This study measured the strip footing near a sandy slope. The slope was loaded centrally and randomly. Ultimate bearing capacity decreased by increasing the amount of eccentricity, and percentage of this decrease increased

with increasing eccentricity. Depending on the soil's relative density, there are generally three shear-failure mechanisms under shallow foundations. Foundations on dense sand ($Dr > 70\%$) fail with a mechanism marked by a peak resistance, which is known as general shear failure [23]. On sands with a relative density between 35% and 70 % sudden failure is not realized. This type of failure is called local shear failure. Foundations placed on very loose sand with $Dr < 35\%$ can penetrate into the soil with no bulging observed on the surface; this failure type is also called punching shear [24]. Soils with relative densities over 70% show a general shear failure mechanism under shallow foundations [25]. Fig.1 shows three shear-failure mechanisms for soil under shallow foundations. This figure shows that the punching mechanism of the soil moves downward directly below the foundation, but the local and general failure mechanisms can facilitate movement to the surface of the soil [25, 26].

The particle image velocimetry (PIV) method explains the sandy soil's movement. It measures the soil particle movement for the whole soil mass [27, 28]. Slominski [29] used PIV method to measure dry cohesion-free sand movement in silos. The surface deformation in silos was studied in laboratory model tests, and effect of the roughness of silo walls and sand density on the volumetric strain was reported. The accuracy of the measurements was discussed and advantages and disadvantages of the PIV method were outlined here. Ould Baba and Peth [30] studied the creep deformation of slopes with a large-scale soil box via PIV. The efficiency of the PIV method was examined, and effect of hydraulic stresses on the creep deformation was studied using model tests.

There are many studies that use strip footing or particle properties, but the effect of particle roundness and shape on the soil behavior and the failure mechanism of soil are missing from the literature. This study is novel because it measures the effect of particle shape and roundness on the soil behavior under strip footings. The failure mechanism of the aggregate was studied with the PIV method at different values of roundness, density, and size.

Figure 1. Failure mechanism of soil under shallow foundations: (a) general shear failure (b) local shear failure (c) punching shear failure [25, 26].

2 MATERIAL AND METHODS

2.1 Soil

The soil was calcareous and prepared by the Ergunler Company in Erzurum, Turkey. According to ASTM 854-14, [31] the soil's specific gravity was 2.7; soil was angular. After taking the soil to the laboratory, it was washed and dried by spreading on a dry surface at room temperature and then sieved to six different sizes

between 0.30 mm to 4.75 mm. Tests were made on the angular, rounded and well-rounded soils. The angular calcareous soil changed to sub-rounded and rounded calcareous. It can be changed to well-rounded calcareous soil via the Los Angeles Rattler machine. Here, angular soil was transformed in the Los Angeles machine without balls, as explained by Arasan [32, 39]. To change the angular calcareous soil to rounded soil, 50,000 revolutions were used. To change the angular soil to well-rounded calcareous soil, 100,000 revolutions were used on the Los Angeles machine. The roundness values

of the soils were determined using the Cox equation [33] and the Power [34] chart (Table 1).

According to the unified soil-classification system [35], soil samples were classified as poorly graded sand (SP). Grain size distribution increased according to ASTM-D 6913-04 [36], as shown in Fig. 2.

The roundness of the soil particles increased by going from angular soil towards rounded and well-rounded. It is even clear and easily visible by eye at larger sizes. Examples of the angular, rounded and well-rounded calcareous soils are shown in Fig. 3.

Table 1. Roundness properties of materials [7, 39].

Sample	Roundness Value	Roundness Class
Angular Calcareous	0.693-0.744	Angular - Sub Angular
Rounded Calcareous	0.786-0.803	Rounded - Well Rounded
Well-Rounded Calcareous	0.834-0.854	Well Rounded

2.2 Model Tank

This study involved a series of laboratory tests. The model tests were conducted in a model tank: 1 m × 0.1 m in plan and 1 m in depth. The tank was completely rigid to prevent any movement and the length of the model footing was almost equal to the width of the tank to maintain plain-strain conditions. The front side of

Figure 2. Grain size distributions of the soils.

Figure 3. Pictures of soil with three different roundness.

the tank was made of a sheet of Plexiglas to monitor and inspect the soil and model the footing and their movements during the tests. A polyamide strip footing was used: 9.8 cm long, 5 cm wide and 4 cm high. It was sufficiently rigid to prevent reshaping during tests. A hydraulic jack was fixed to a strong horizontal beam of the frame that could carry thrust developed by hydraulic jack without any deformation during tests. The speed of displacement was <2 mm/min for applying loads over small increments. A 50-kN load cell was placed between the jack and footing to measure the applied load. The load was transferred to the footing via a shaft placed between the load cell and the footing. A ball bearing was placed between the shaft and the footing to apply a single point load to the footing. A rigid footing was used in this study, and a uniform load was applied from the footing to the soil. There was a 1-mm gap at each side of the tank to prevent contact between the side walls of the tank and the footing. Two sides of the footing and walls of the tank were coated with petroleum jelly to reduce the end-friction effects.

Two linear variable displacement transducers (LVDTs) were concurrently placed at the two corners of the model footing. The average movement of these two LVDTs was considered to be settlement of the footing.

Finally, the data from the load cell and the LVDTs were transferred to a computer via a data logger. To analyze the soil movement, a high-resolution digital camera was placed in front of the tank to take high-quality images. First, a picture was taken before starting the test without any movement in the soil or the footing. Images were acquired every 30 seconds until the end of the experiment. These pictures were used to analyze the

Figure 4. Pictures of the tank.

Figure 5. Schematic picture of the test box.

soil movement via the Geo-PIV program. Tank pictures and a schematic drawing of the model tank are shown in Figs. 4 and 5, respectively.

2.3 Experimental Setup

The soil was classified into six different sizes from 0.3 mm to 4.75 mm. There were three roundness classes: angular, rounded and well-rounded calcareous for each dimension. The tests were done at relative densities of 30%, 50% and 70% ($Dr=30\%$, 50% and 70%) for each group of dimensions and roundness classes. According to ASTM D4253-16 and ASTM D4254-16, [37, 38] the minimum and maximum void ratio (e_{min} and e_{max}) of each soil sample was identified by considering the tank volume. The weight of the soil was calculated and placed in the tank at these three densities. There were 54 samples for testing, and each test was done at least in triplicate to ensure the results. Table 2 shows all the test conditions. After putting the soil inside the tank, a strip footing was placed on the soil surface, and two LVDTs at two cross corners of the footing were used to measure the settling of the model foundation.

Table 2. All 54 different conditions of the tests.

Soil Dimension (mm)	Relative Density (%)	Roundness Class
0.30–1.18	30 - 50 - 70	Angular, Rounded, Well-Rounded
1.18–1.40	30 - 50 - 70	Angular, Rounded, Well-Rounded
1.40–2.36	30 - 50 - 70	Angular, Rounded, Well-Rounded
2.36–2.80	30 - 50 - 70	Angular, Rounded, Well-Rounded
2.80–3.35	30 - 50 - 70	Angular, Rounded, Well-Rounded
3.35–4.75	30 - 50 - 70	Angular, Rounded, Well-Rounded

2.4 PIV Method

The Geo-PIV program can explain the mechanism of soil deformation under the footing. PIV is a velocity-measurement technique originally developed in the field of fluid mechanics. Soil deformation can be considered a low-velocity flow process [28]. This program is based on close-range photography and PIV. In Geo-PIV, digital photographs of planar soil deformation are studied with PIV to monitor the movement of the soil particles. To measure the soil movement, the program creates patches

on the pictures and compares each patch across the series of pictures. This shows the final movement of the particles for each patch as well as the whole soil [27]. Here, a digital camera is placed in front of the tank on a stable tripod to prevent any movement. The first picture was taken before starting the test. This explains the soil condition before adding any load. The second picture was taken 30 seconds after loading the footing, and then a series of pictures were taken every 30 seconds to the end of the test. All the pictures are high resolution, and there was no movement of the camera or the tank during the testing.

3 RESULTS AND DISCUSSIONS

The tests were made under laboratory conditions, and high-quality pictures were collected during the testing. Data from the load-cell and the LVDTs were transferred to the computer and a settlement-load graph was drawn for each test. These graphs show a failure point and a subsequent failure moment that is recognized for each test [24, 25]. Next, the pictures' step numbers were identified by considering the failure time. This series of pictures was analyzed with Geo-PIV software to understand the soil movement and the deformation from the beginning to the failure point. This was done for each test to evaluate the shear failure of the soil under the footing. Fig.6 shows the failure mechanism of the angular calcareous soils at six different dimensions and three different relative densities (Dr ; 30%, 50%, and 70%). The data suggest that the soil-failure mechanism goes from punching to general failure by increasing the relative density. The shear failure mechanism is punching at a 30% relative density. At $Dr=50\%$, this mechanism changed to local. By increasing the Dr to 70%, the soil-failure mechanism became a general shear failure.

The movement and deformation increase with the increasing soil size. The underlying layers of soil deform at dimensions of 0.30–1.18 mm near 1.5 B. At $Dr=30\%$, the 2B samples have a soil size of 3.35–4.75 mm at the same relative density. The side areas of the footing showed the same behavior on larger sizes. There is more area of soil under the footing. The inside soil was affected by the loading and even the soil mechanism went from punching to a general mechanism. By increasing the relative density of the soil, the influence of the aggregate size was more pronounced – especially on the side movements. More areas under the footing showed evidence of deformation. The deformation at larger sizes was more pronounced than with the finer soils.

Figure 6. Failure mechanism of angular soil at different sizes and relative densities.

We also studied the effect of roundness on the soil-failure mechanism. The soil-failure mechanism was compared at three different relative densities and three roundness classes for all six dimensions. The soil movements under these conditions are seen in Fig.7 to 12. These mechanisms showed that by increasing the

roundness of the material there were decreases on the loading area under the footing. This was more visible for materials with dimensions of 0.30–1.18 than for those of 3.35–4.75 mm. This means that the effect of the roundness is more significant for finer materials than for larger materials.

Figure 7. Failure mechanism of soils with dimensions of 0.30–1.18mm.

Figure 8. Failure mechanism of soils with dimensions of 1.18–1.40mm.

Figure 9. Failure mechanism of soils with dimensions of 1.40–2.36mm.

Figure 10. Failure mechanism of soils with dimensions of 2.36–2.80mm.

The soil behavior under three roundness conditions was different. Angular soils had punching failure at $Dr=30\%$, local shear failure at $Dr=50\%$, and general shear failure at $Dr=70\%$. However, the soil behavior under the footing changed by increasing the particle roundness. The movement of the well-rounded soil was between punching

and local at $Dr=70\%$ with dimensions of 0.30–1.18 mm. Even at dimensions of 3.35–4.75 mm, well-rounded samples failed with a local shear failure mechanism at the same dimension and relative density. This is in contrast to the general failure mechanism of angular soil at a Dr of 70%.

Figure 11. Failure mechanism of soils with dimensions of 2.80–3.35mm.

Figure 12. Failure mechanism of soils with dimensions of 3.35–4.75mm.

Particles can move on each other more easily in well-rounded soils than rounded or angular samples because of the shapes and roundness of the aggregates. The particles in angular soil interlocked causing the soil to act like a continuous area. This is because of the sharp corners in the particles and the higher friction. However, in rounded and

well-rounded materials, the soil particles can move more easily against each other. This caused movement and deformation under the footing. Changing the roundness of the soil particles affected the bearing capacity of the soil and the settlement of the footing. There was a decreased soil bearing capacity associated with the increasing roundness.

4 CONCLUSIONS

The soil behavior changes completely as a function of roundness. Both the particle shape and the roundness should be considered during engineering. This effect is more significant for finer materials. Beneath the affected layers of soil, angular particles are deeper than the rounded and well-rounded particles. These particles should be studied carefully. Angularity has an effect on the soil properties that change the soil's behavior at a distance of B (B is the footing width) from the footing edges. This affects the soil's behavior and should be studied in addition to the soils under the footing – even at a relative density of 30% for the angular soils. Increasing the roundness of the particles can affect the footing and the soil behavior because it depends strongly on the properties of the soil, especially under the footing. The larger relative densities of the angular soils with at least a depth of $4B$ should be studied carefully, because even at this distance under the footing there is deformation and movement of the soil particles.

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